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## Damping of vibration-damping thin-walled steel structures with discrete rubber inserts

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### Abstract

The results of theoretical and experimental studies of the efficiency of noise reduction from excited steel thin-walled structures, modernized with the help of discrete vibration-damping inserts, are presented. As part of the study, an analysis of the factors of dissipation of vibration energy of a plate with inserts was performed. A scheme for placing vibration-damping discrete inserts in the structure under study has been developed. To determine the wave resistance of the damping insert in the plate, a design scheme is adopted. It is hypothesized that during the propagation of flexural waves, the process of dissipation of flexural vibrational energy in a plate with discrete rubber inserts is determined by dry and viscoelastic friction between the elements of the structure under study. It is shown that the process of dissipation of vibrational energy in the plate under study is the same as in a system of excited plates with individual anti-vibration blocks located on its surface. When developing a mathematical model of noise reduction from an oscillating plate with discrete vibration-damping inserts, the necessary suggestions and assumptions were made. The results of experimental studies are presented in the form of graphical dependences of the change in the loss coefficient and the decrease in the sound pressure level in the plate under study, which has damping discrete inserts for various variable parameters. The research results can be used on production equipment for noise reduction and ensuring normal conditions labor on noise factor.

**Keywords:** excited thin-walled steel structures, noise reduction, vibration-damping inserts, excitation frequency, noise, loss coefficient.

### *Демпфирование дискретными резиновыми вставками вибродемпфирующих тонкостенных металлических конструкций*

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### Аннотация

Приведены результаты теоретических и экспериментальных исследований эффективности шумоподавления от возбужденных металлических тонкостенных конструкций, модернизированных

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с помощью дискретных вибродемпфирующих вставок. В рамках исследования выполнен анализ факторов диссипации энергии колебаний пластины со вставками. Разработана схема размещения в исследуемой конструкции вибродемпфирующих дискретных вставок. Для определения волнового сопротивления демпфирующей вставки в пластине принята расчетная схема. Выдвинута гипотеза, что при распространении изгибных волн процесс диссипации изгибной колебательной энергии в пластине с дискретными резиновыми вставками определяется сухим и вязкоупругим трением между элементами исследуемой конструкции. Показано, что процесс диссипации колебательной энергии в исследуемой пластине такой же, как и в системе возбужденных пластин с штучными антивибрационными блоками, расположенными на ее поверхности. При разработке математической модели шумоподавления от колеблющейся пластины с дискретными вибродемпфирующими вставками были приняты необходимые предположения и допущения. Приведены результаты экспериментальных исследований в виде графических зависимостей изменения коэффициента потерь и снижения уровня звукового давления в исследуемой пластине, имеющей демпфирующие дискретные вставки при различных переменных параметрах. Результаты исследований могут быть использованы на производственном оборудовании для шумоподавления и обеспечения нормальных условий труда по шумовому фактору.

**Ключевые слова:** возбуждённые тонкостенные металлические конструкции, шумоподавление, вибродемпфирующие вставки, частота возбуждения, шум, коэффициент потерь.

## Introduction

Let us consider the problem of noise reduction during vibration excitation of thin-walled steel structures (TSS). To reduce the transmission of vibration and noise, vibration isolation materials and devices, such as rubber gaskets, shock absorbers and soundproofing materials, can be used. These methods make it possible to isolate TSS from the environment and reduce sound transmission.

Per sound power ( $W$ ), emitted by a plate TSS are affected by the oscillatory speed  $v^2$  and plate area  $S$ .

Effective protection against noise created by a vibrating TSS are damping methods that affect the above parameters. Among damping methods, the most common is vibration damping coatings (VDC) [1-4].

VDC are one of the effective means for reducing noise from excited TSS. VDC can reduce the amplitude of vibrations of the TSS surface, which can lead to a decrease in the surface area of emitted sound waves. This occurs due to the absorption of part of the vibrational energy. VDC can reduce the speed of oscillations of TSS due to the absorption and dissipation of oscillation energy. This leads to a decrease in the frequency and amplitude of the oscillations, which in turn reduces the frequency and amplitude of the emitted noise.

However, the VDC does not fully meet the conditions of the task, that is, reducing the noise level with the help of special TSS. Therefore, we propose the design of a rubber piece vibration-damping insert (PVI), which is fixed in the perforation of a steel plate. Rubber gaskets generally withstand a variety of environmental conditions such as humidity and temperature changes.

A diagram with a discrete vibration-damping insert placed in the perforation hole of the plate is shown in Figure 1.

Using loss factor ( $\eta$ ) vibrational energy describes the dissipative properties of the VDC. Energy is generated in the TSS in the event of oscillatory movements. The properties of VDC are considered in studies [1-10].

The loss coefficient is used when assessing the dissipative properties of VDC and other

materials used to reduce vibration and noise in engineering systems. The higher the loss coefficient value, the more significantly the material suppresses vibrations and reduces sound power. When selecting a VDC for a specific application, it is important to consider the loss factor and compare it with the noise reduction and attenuation requirements of the system.

Existing mathematical models describing the process of dissipation of vibration energy of plates with various types of coatings (soft, hard, reinforced) are quite sufficient and informative. According to these mathematical models, the loss coefficient ( $\eta_{\Sigma}$ ) can be determined either in the structure of the coated object under study, or as the loss coefficient of the coating material, if  $\eta_{\Sigma}$  was established experimentally.

Expected effect of reducing TSS noise from vibration excitation:

$$\Delta L = 20 \lg \frac{\eta_{\Sigma}}{\eta_1},$$

where  $\eta_1, \eta_{\Sigma}$ , – the total loss coefficients of the plate before and after coating, respectively.

To calculate a specific noise reduction effect, it is necessary to use equations related to material and structural losses for engineering calculations. To do this, you can use formulas to estimate the loss coefficient and take into account the nature of the deformation of the coating.

For plates with vibrational excitation with PVI, there is no theoretical understanding of the dissipation of vibrational energy.

A hypothesis is proposed according to which the dissipation of vibrational energy in the 'plate–PVI' system is a consequence of a combination of factors:

- the energy of the longitudinal wave is reduced by adding a PVI to the plate, which in turn acts as an obstacle during the propagation of such a wave;
- dry friction occurs between the edge of the perforated hole of the plate and the surface of the welding joint;
- viscoelastic friction occurs in the body of the ball-and-roll motor made of rubber [11-13].

Physically, the process of energy dissipation of an excited plate with PVI largely coincides with the physical picture in a system of plates with oscillatory excitation with a partial local anti-vibration block, consisting of local components on its surface. In the study [9], the antivibrator is considered as a local oscillatory system having certain mass, friction and elastic components (Figure 2).

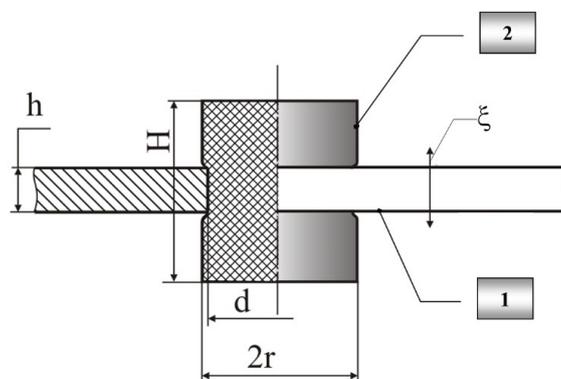


Fig. 1. Schematic illustration of wafer placement damping insert:  
1 – metal plate; 2 – rubber damping insert

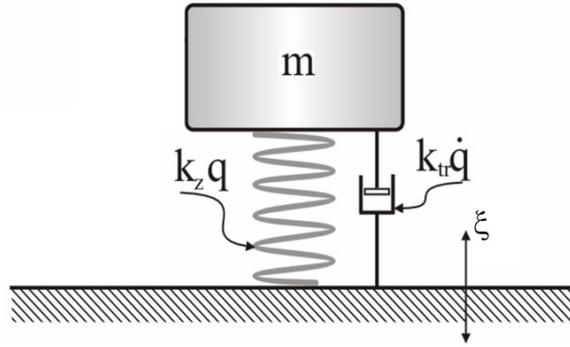


Fig. 2. Design diagram of a local antivibrator

### 1. 1. Mathematical model of noise reduction of an oscillating plate with discrete vibration-damping inserts

Figure 3 shows a plate with discrete vibration-damping rubber inserts.

To determine the numerical value of the drop in the sound pressure level of a plate equipped with shock-absorbing discrete rubber inserts, we will use the method of wave impedance of thin plates.

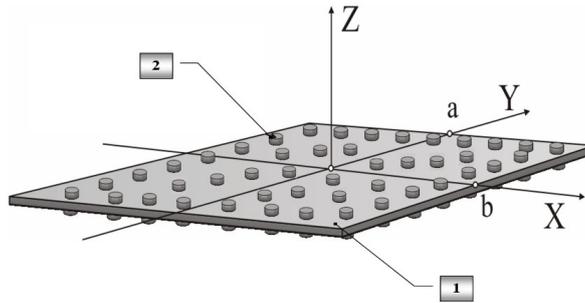


Fig. 3. Plate with vibration-damping inserts: 1 – metal plate; 2 – discrete rubber inserts

To determine the numerical value of the drop in the sound pressure level of a plate equipped with shock-absorbing discrete rubber inserts, we will use the method of wave impedance of thin plates.

Next, we denote:

$N$  – the number of damping inserts;

$\rho_r$  – the density of the damping rubber insert material,  $kg/m^3$ ;

$m$  – the weight of one damping rubber insert,  $kg$ ;

$\rho$  – the density of the plate material,  $kg/m^3$ ;

$G$  – dynamic shear modulus,  $N/m^2$ ;

$\rho_w$  – air density,  $kg/m^3$ ;

$c$  – the speed of sound in the air,  $m/s$ ;

$h$  – plate thickness,  $m$ ;

$k_z$  – coefficient of rigidity of the insert material,  $N/m$ ;

$H$  – height of the damping insert,  $m$ ;

$d$  – diameter of the hole for the damping insert,  $m$ ;

$q$  – oscillatory movements of the damping insert,  $m$ ;

$x, y$  – coordinate axes of the plate,  $m$ ;

$\gamma$  – the dimensionless area of the plate, expressing the ratio of the actual area of the

plate to the unit area;

$\nu$  – Poisson's ratio;

$\omega$  – oscillation frequency,  $s^{-1}$ ;

$E$  – Young's modulus,  $N/m^2$ ;

$\xi$  – transverse displacement of the plate,  $m$ ;

$k$  – wave number of bending waves;

$F_o$  – amplitude of transverse forces on a plate of unit area,  $N/m^2$ ;

$F_a$  – the resistance force of the ambient air, related to the unit area of the plate,  $N/m^2$ ;

$Z_1, Z_2, Z_3$  – the wave resistance of the plate, the damping insert, the plate with damping inserts, respectively;

$W_0$  – the energy radiated and absorbed by the plate with damping inserts for half the oscillation period,  $W$ ;

$W_0$  – energy in the plate with damping inserts,  $W$ ;

$B$  – bending stiffness of the plate;

$D$  – complex bending stiffness of the plate;

$\eta_1, \eta_2, \eta_\Sigma$  – loss coefficients of the plate, damping insert, plate with damping inserts (total loss coefficient), respectively;

$\Delta L$  – change in sound pressure level,  $dB$ ;

$i = \sqrt{-1}$  – an imaginary unit.

We describe the process of changing the sound pressure level in the air using the expression [7]:

$$\Delta L(\omega) = 20 \lg \left( \frac{\eta_\Sigma(\omega)}{\eta_1(\omega)} \right). \quad (1)$$

We will determine the loss coefficient using the Kirchhoff–Love hypothesis, according to which the following assumptions are introduced:

- we consider the infinitesimal element of the plate to retain its length and straight line, as well as the normal to the central plane;

- small elastic transverse deformations take place in the plate, residual deformations do not occur;

- external forces cause a flat stressed state in the plate;

- when the plate is bent in the middle surface, deformations do not occur.

The internal forces in the plate are determined from the expressions:

$$\left. \begin{aligned} M_{11} &= B \left( \frac{\partial^2 \xi}{\partial x^2} + \nu \frac{\partial^2 \xi}{\partial y^2} \right); M_{22} = B \left( \frac{\partial^2 \xi}{\partial y^2} + \nu \frac{\partial^2 \xi}{\partial x^2} \right); \\ M_{12} = M_{21} &= B(1 - \nu) \frac{\partial^2 \xi}{\partial x \partial y}; Q_1 = B \frac{\partial}{\partial x} \Delta \xi; Q_2 = B \frac{\partial}{\partial y} \Delta \xi; \end{aligned} \right\}, \quad (2)$$

where  $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  – Laplace operator.

Bending vibrations are determined from the expression:

$$\Delta(B\Delta\xi) - (1 - \nu) \left[ \frac{\partial^2}{\partial x^2} \left( B \frac{\partial^2 \xi}{\partial y^2} \right) + \frac{\partial^2}{\partial y^2} \left( B \frac{\partial^2 \xi}{\partial x^2} \right) - 2 \frac{\partial^2}{\partial x \partial y} \left( B \frac{\partial^2 \xi}{\partial x \partial y} \right) \right] + \rho h \frac{\partial^2 \xi}{\partial t^2} = F(x, y, t)$$

In the case where the plate thickness and , the vibration equation of the plate will take the form:

$$B\Delta\Delta\xi + \rho h \frac{\partial^2 \xi}{\partial t^2} = F(x, y, t) \quad (3)$$

The force acting on the plate per unit area consists of the excitation forces and air resistance.

The one-dimensional wave equation has the form:

$$\frac{d^2\psi}{dz^2} - \frac{1}{c^2} \frac{d^2\psi}{dt^2} = 0, \quad (4)$$

where  $\psi$  – speed potential of an air medium having a density  $\rho_w$ , having a pressure increment  $p$  and the speed of acoustic waves  $V$ :

$$p = -\rho_w \frac{d\psi}{dt}, V = \frac{d\psi}{dz}. \quad (5)$$

At  $\dot{\xi}(t) = i\omega\xi e^{i\omega t}$  the potential can be written in the form

$$\psi(z,t) = \psi_0 e^{(\lambda z + i\omega t)}$$

and the solution to equation (4) will take the form:

$$\left(\lambda^2 + \frac{\omega^2}{c^2}\right) \psi_0 e^{i\omega t} = 0$$

or

$$\psi = A_1 \exp\left(i\omega\left(t + \frac{z}{c}\right)\right) + B_1 \exp\left(i\omega\left(t - \frac{z}{c}\right)\right). \quad (6)$$

Based on equations (5) and (6) we obtain:

$$F_a = \rho_w c \dot{\xi}(x,y,t). \quad (7)$$

The exciting force is determined by the formula

$$F_1(x,y,t) = F_0 \exp(i(\omega t - k(x+y))), \quad (8)$$

and taking into account equation (7), we transform (3) to the form:

$$D \left( \frac{\partial^4 \xi}{\partial x^4} + 2 \frac{\partial^4 \xi}{\partial x^2 \partial y^2} + \frac{\partial^4 \xi}{\partial y^4} \right) + \rho h \frac{\partial^2 \xi}{\partial t^2} = F_0 \exp(i(\omega t - k(x+y))) - \rho_w c \frac{\partial \xi}{\partial t}, \quad (9)$$

Let's transform (9) to the form:

$$\xi(x,y,t) = \xi_0 \exp(i(\omega t - k(x+y))), \quad (10)$$

or

$$\frac{\partial}{\partial t} \xi(x,y,t) = \dot{\xi} = \xi_0 i \omega \exp(i(\omega t - k(x+y))). \quad (11)$$

Let us substitute (10) into (9), perform differentiation, take (11) into account, and obtain the expression:

$$D4k^4 \xi_0 e^{i(\omega t - k(x+y))} + \rho h i \omega \xi_0 i \omega e^{i(\omega t - k(x+y))} = F_0 e^{i(\omega t - k(x+y))} - \rho_w c \xi_0 i \omega e^{i(\omega t - k(x+y))}. \quad (12)$$

Flexural stiffness is calculated from the expression:

$$D = \frac{(1 + i\eta)Eh^3}{12(1 - \nu^2)}. \quad (13)$$

Based on (12) we determine:

$$z_1(\omega) = \frac{F_0}{\xi_0 i\omega} = \frac{4Dk^4}{i\omega} + \rho h i\omega + \rho_w c. \quad (14)$$

Transforming (14), we obtain:

$$z_1(\omega) = 4\eta \frac{Eh^3}{12(1 - \nu^2)} \frac{k^4}{\omega} + \rho_w c + i \left[ \rho h \omega - 4 \frac{Eh^3}{12(1 - \nu^2)} \frac{k^4}{\omega} \right], \quad (15)$$

where  $k = \sqrt{\frac{\omega}{r \cdot c_n}}$  – wave number,  $r = \frac{h}{\sqrt{12}}$  – moment of inertia of the section of a plate of unit area,  $c_n = \sqrt{\frac{E}{\rho(1 - \nu^2)}}$  – bending wave phase velocity.

Let's divide the expression into imaginary and real parts:

$$\left. \begin{aligned} Re(z_1(\omega)) &= \frac{4B\eta}{(r \cdot c_n)^2} \omega + \rho_w c \\ Im(z_1(\omega)) &= \left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega \end{aligned} \right\}, B = \frac{Eh^3}{12(1 - \nu^2)}. \quad (16)$$

We will write down the loss coefficient of the plate under the condition of damping of the surrounding air:

$$\eta_1(\omega) = \frac{W_0}{\pi \cdot W_{\dot{r}}} = \frac{Re(z_1(\omega))}{|z_1(\omega)|} = \frac{1}{\sqrt{1 + \left[ \frac{Im(z_1(\omega))}{Re(z_1(\omega))} \right]^2}}$$

or

$$\eta_1(\omega) = \frac{1}{\sqrt{1 + \left[ \frac{\left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega}{\frac{4B\eta}{(r \cdot c_n)^2} \omega + \rho_w c} \right]^2}}. \quad (17)$$

We use equation (17) to construct a graph (Figure 4).

To determine the physical parameters of the damping insert, the main parameter of the damping insert is determined. To determine the parameters of plate motion, the oscillatory motion of the plate is studied. This includes the amplitude, frequency and mode of vibration of the plate. Determining the wave properties of a medium includes the study of wave impedance, which depends on the type of medium through which the wave caused by the vibration of the plate passes.

The calculation of wave resistance is performed using the obtained data on the movement of the plate and the properties of the medium; it is possible to additionally calculate the wave resistance of the damping insert. This may require special equations and calculation methods, depending on the specific conditions of the problem. After the calculation and determination of the wave resistance value has been completed, it is necessary to analyze the results. Evaluate how the characteristic impedance of the insert affects the vibration of the plate and how effectively it absorbs or reduces vibration. The calculation scheme for determining the wave resistance is shown in Figure 2.

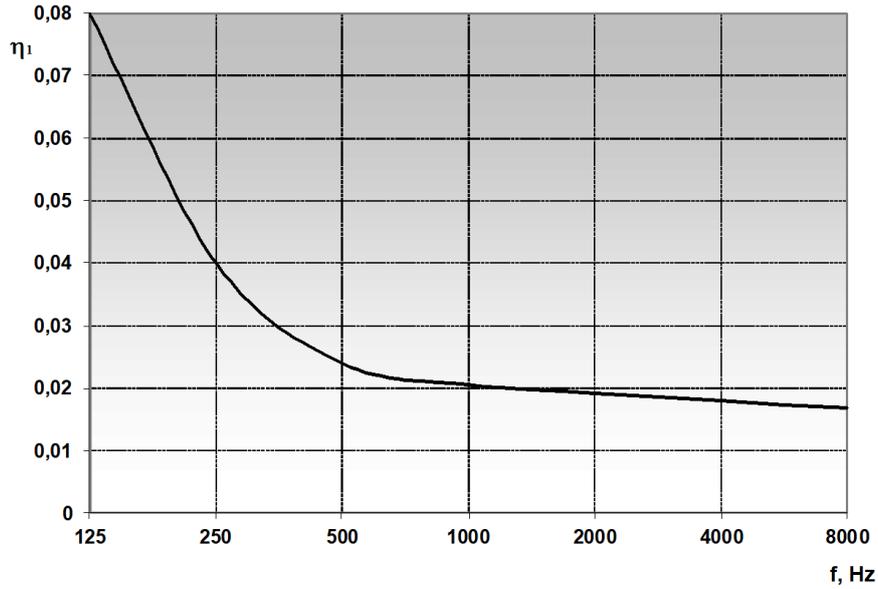


Fig. 4. Graph of loss coefficient values in the plate on the excitation frequency at the initial data:  $h = 0,0025$  m,  $E = 2,1 \cdot 10^7$  N/m<sup>2</sup>,  $\nu = 0,3$ ,  $\rho = 7850$  kg/m<sup>3</sup>,  $\rho_w = 1,29$  kg/m<sup>3</sup>,  $c = 330$  m/s,  $\eta = 0,015$

The force of elastic deformation can be taken into account using the stiffness coefficient:

$$k_z = Gh. \quad (18)$$

For the source of oscillatory motion of the damping insert, the following law is valid:

$$\xi(\omega, t) = \xi_0 e^{i\omega t}, \quad (19)$$

where  $\xi_0$  – amplitude of transverse movement of the plate.

This law may differ for different scenarios and types of plate vibration. To take into account the influence of lateral displacement on the oscillatory movement of the damping insert, a mathematical description of the movement of the plate should be performed, that is, a mathematical description of the movement of the plate taking into account the lateral displacement. This may require an equation for the vibration of the plate, which depends on time and coordinates.

Determining the force acting on the insert using a mathematical description of the movement of the plate, determining which parts of the movement affect the insert when it is damped. This may involve breaking down the displacement into components and determining how each component affects the elastic force and the friction force. Calculation of wave resistance is used when obtaining data on forces (elastic deformation force and friction force) to calculate the wave resistance of the damping insert. Analysis of the results involves consideration of the obtained values of wave resistance and its influence on the oscillatory motion of the damping insert. This analysis allows us to understand how the insert effectively suppresses vibration, taking into account the lateral movement of the plate.

The equation of motion of the damping insert has the following form:

$$m\ddot{q} + k_{tr}(\dot{q} - \dot{\xi}) + k_z(q - \xi) = 0, \quad (20)$$

where  $m = H\pi \cdot r^2 \rho_r$  – weight of one damping insert.

The oscillatory motion of the plate occurs with a certain frequency, which is also transmitted to the damping insert, and their frequencies coincide in stationary mode.

The vibrational motion of the plate can be described by a parameter such as the vibration frequency (set frequency), that is, the frequency at which the board oscillates or vibrates. It is determined in relation to the magnitude of the external influence causing vibration of the board. The damping insert allows you to control the amplitude and duration of vibration of the plate. It is usually used to reduce vibration energy and keep the system in a less mobile state. Damping can occur in various ways, for example through energy loss within the material or through damping elements.

Stationary mode is a state of the system in which the amplitude and frequency of vibration of the plate remain constant over time. It is achieved when the energy entering the system is equal to the energy leaving the system as a result of attenuation and other losses. If the set vibration frequency of the board is the same as the frequency in stationary mode, this may mean that the system is in a resonant state, which can lead to an increase in vibration amplitude. Controlling this process is important to prevent system damage or ineffective operation.

Let us transform expression (19) to the form:

$$q(\omega, t) = q_0 e^{i(\omega t)}, \quad (21)$$

where  $q_0$  – amplitude of oscillatory movement of the insert

Solving jointly (21), (20), (19):

$$m\ddot{q} + k_{tr}\dot{q} + k_z q = (k_{tr}i\omega + k_z)\xi_0 e^{i\omega t},$$

or

$$\frac{(m\ddot{q} + k_{tr}\dot{q} + k_z q)}{(k_{tr}i\omega + k_z)} = \xi_0 e^{i\omega t}. \quad (22)$$

We find the transfer force of inertia from the expression:

$$F_i = -m\ddot{\xi}(\omega, t) = F_0 e^{i\omega t},$$

where

$$F_0 = m\omega^2 \xi_0. \quad (23)$$

Damping insert impedance:

$$z_2(\omega) = \frac{F_0}{\dot{q}_0} = \frac{m\omega^2 \xi_0}{i\omega q_0}. \quad (24)$$

Let us transform (22) to the following form:

$$\begin{aligned} \frac{m\omega^2 e^{i\omega t}}{(k_{tr}i\omega + k_z)} \left( -\frac{m\omega}{i} i\omega q_0 + k_{tr}i\omega q_0 + \frac{k_z}{i\omega} i\omega q_0 \right) &= F_0 e^{i\omega t}, \\ z_2(\omega) &= [(m\omega^2 - k_z)i + k_{tr}\omega] \frac{m\omega}{k_{tr}i\omega + k_z}, \\ z_2(\omega) &= \frac{k_{tr}m^2\omega^4}{k_z^2 + k_{tr}^2\omega^2} + i \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2 m\omega^3}{k_z^2 + k_{tr}^2\omega^2}. \end{aligned} \quad (25)$$

The real and imaginary parts are as follows:

$$Re(z_2(\omega)) = \frac{k_{tr}m^2\omega^4}{k_z^2 + k_{tr}^2\omega^2},$$

$$Im(z_2(\omega)) = \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2m\omega^3}{k_z^2 + k_{tr}^2\omega^2}.$$

Loss factor for a single rubber insert:

$$\eta_2(\omega) = \frac{Re(z_2(\omega))}{|z_2(\omega)|} = \frac{1}{\sqrt{1 + \left[ \frac{Im(z_2(\omega))}{Re(z_2(\omega))} \right]^2}}$$

or

$$\eta_2(\omega) = \frac{1}{\sqrt{1 + \left[ \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2m\omega^3}{k_{tr}m^2\omega^4} \right]^2}}. \quad (26)$$

Using equation (26), we plot the graph of the relationship of the loss coefficient values for the rubber insert (Figure 5).

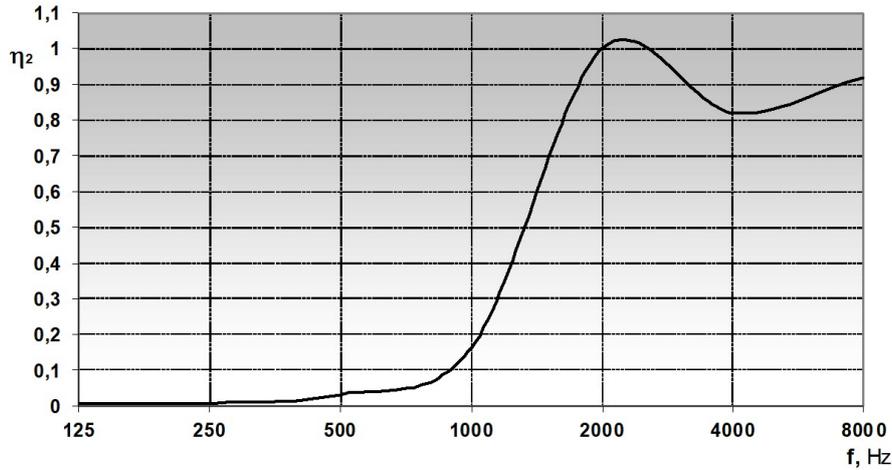


Fig. 5. Graph of the relationship between the loss coefficient values in the rubber insert and the excitation frequency in the plate with initial data:

for the plate –  $h = 0,0025$  m,  $d = 0,01$  m; to insert –  $G = 9,6 \cdot 10^5$  N/m<sup>2</sup>,  $\rho = 1400$  kg/m<sup>3</sup>,  
 $k_z = 2,4 \cdot 10^3$ ,  $k_{tr} = 1,1$ ,  $m = 1,583 \cdot 10^{-3}$  kg

The characteristic impedance of the composite plate is determined from the expression

$$z_3(\omega) = \gamma \cdot z_1(\omega) + N \cdot z_2(\omega).$$

From (15) and (25) taking into account  $\gamma = \frac{4ab}{1}$ , we obtain:

$$z_3 = \frac{\gamma 4B\eta}{(r \cdot c_n)^2} \omega + \gamma \rho_w c + \frac{N k_{tr} m^2 \omega^4}{k_z^2 + k_{tr}^2 \omega^2} + i \left\{ \gamma \left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega + N \left[ \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2 m \omega^3}{k_z^2 + k_{tr}^2 \omega^2} \right] \right\}.$$

The total loss coefficient in the plate is obtained by separating the imaginary and real parts in the above equation, as a result we obtain:

$$\eta_{\Sigma}(\omega) = \frac{\operatorname{Re}(z_3(\omega))}{|z_3(\omega)|} = \frac{1}{\sqrt{1 + \left[ \frac{\operatorname{Im}(z_3(\omega))}{\operatorname{Re}(z_3(\omega))} \right]^2}}$$

or

$$\eta_{\Sigma}(\omega) = \frac{1}{\sqrt{1 + \left[ \frac{\gamma \left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega + N \left[ \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2 m \omega^3}{k_z^2 + k_{tr}^2 \omega^2} \right]}{\frac{\gamma 4B\eta}{(r \cdot c_n)^2} \omega + \gamma \rho_w c + \frac{N k_{tr} m^2 \omega^4}{k_z^2 + k_{tr}^2 \omega^2}} \right]^2}}. \quad (27)$$

Figure 6 shows the graph obtained using equation (27).

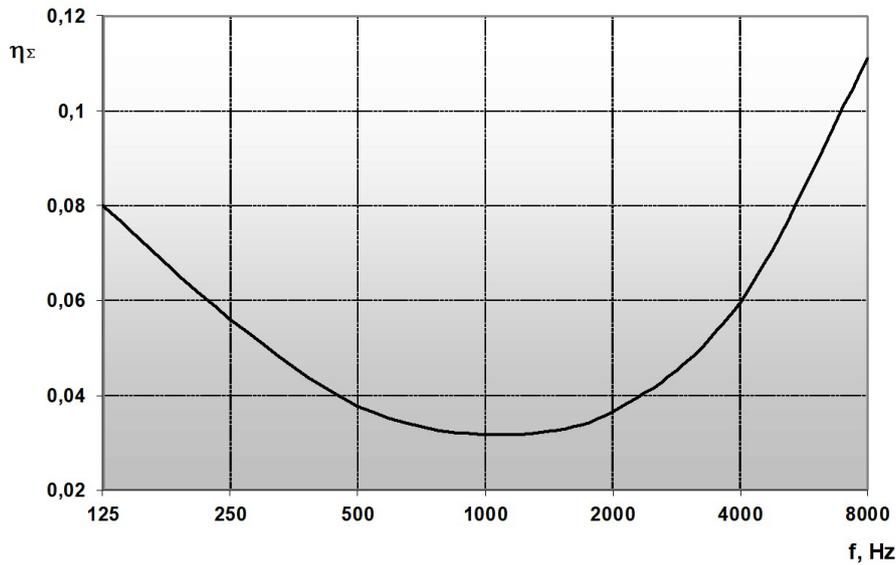


Fig. 6. Graph of the relationship between the values of the total loss coefficient in a plate with a rubber insert and the excitation frequency with initial data: for the plate –  $h = 0,0025$  m,  $d = 0,01$  m;  $\gamma = 0,25$ ; for rubber insert –  $G = 9,6 \cdot 10^3$  N/m<sup>2</sup>,  $\rho = 1400$  kg/m<sup>3</sup>,  $k_z = 2,4 \cdot 10^3$ ,  $k_{tr} = 1,1$ ,  $m = 1,583 \cdot 10^{-3}$  kg,  $N = 80$

The amount of reduction in sound pressure level is obtained from expression (1) and equations (25), (17):

$$\Delta L(\omega) = 20 \lg \left( \frac{1 + \left[ \frac{\left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega}{\frac{4B\eta}{(r \cdot c_n)^2} \omega + \rho_w c} \right]^2}{1 + \left[ \frac{\gamma \left[ \rho h - \frac{4B}{(r \cdot c_n)^2} \right] \omega + N \left[ \frac{k_z(m\omega^2 - k_z)m\omega - k_{tr}^2 m \omega^3}{k_z^2 + k_{tr}^2 \omega^2} \right]}{\frac{\gamma 4B\eta}{(r \cdot c_n)^2} \omega + \gamma \rho_w c + \frac{N k_{tr} m^2 \omega^4}{k_z^2 + k_{tr}^2 \omega^2}} \right]^2} \right), \quad (28)$$

where  $B = \frac{Eh^3}{12(1-\nu^2)}$ ,  $r = \frac{h}{\sqrt{12}}$ ,  $c_n = \sqrt{\frac{E}{\rho(1-\nu^2)}}$ . We take into account that

$$20 \lg(\sqrt{x}) = 4,343 \ln(x),$$

Then we write expression (2.28) in the form:

$$\Delta L = 4,343 \left[ \ln \left( 1 + \frac{9\omega^2}{(4\eta\omega + \frac{\rho_w c}{\rho h})^2} \right) - \ln \left( 1 + \left( \frac{N \frac{m\omega(k_z m\omega^2 - k_z^2 - k_{tr}^2 \omega^2) - 3\gamma\rho h\omega}{k_z^2 + k_{tr}^2 \omega^2}}{N \frac{k_{tr} m^2 \omega^4}{k_z^2 + k_{tr}^2 \omega^2} + \gamma\rho_w c + 4\gamma\rho h\eta\omega} \right)^2 \right) \right]. \quad (29)$$

In Figure 7 graphs of changes in sound pressure level and frequency ( $f$ ) and on the number of rubber inserts are plotted ( $N$ ) [11, 12].

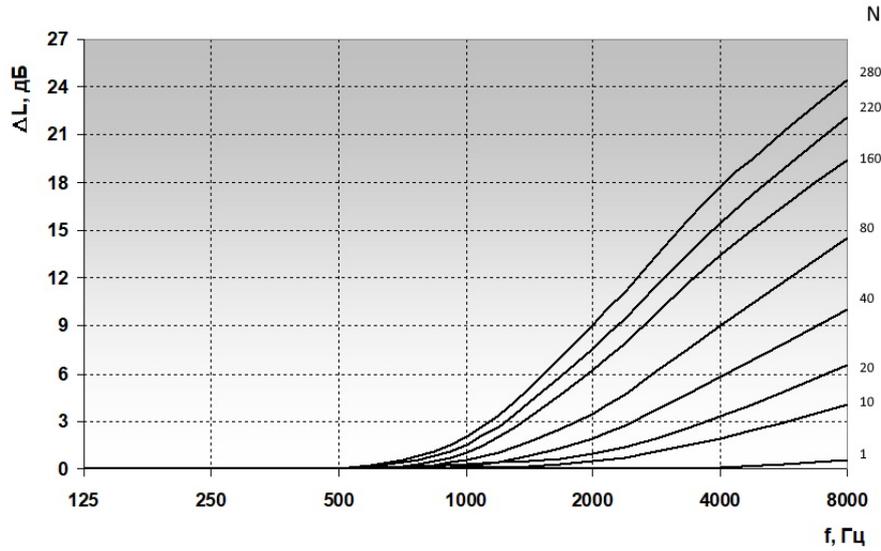


Fig. 7. Graph of the relationship between the sound pressure values of a plate with rubber inserts and the excitation frequency with initial data:

for the plate –  $d = 0,01$  m,  $h = 0,0025$  m,  $\gamma = 0,25$ ; for rubber insert –  $m = 1,583 \cdot 10^{-3}$  kg,  $G = 9,6 \cdot 10^3$  N/m<sup>2</sup>,  $\rho = 1400$  kg/m<sup>3</sup>,  $k_z = 2,4 \cdot 10^3$ ,  $k_{tr} = 1,1$ ,  
 $N = [280, 220, 160, 80, 40, 20, 10, 1]$

The use of equations such as (29) in engineering practice can significantly improve the design and efficiency of damping systems, which are important in many industries where vibration and noise control are critical.

## Conclusions

1) Based on the analysis of theoretical studies, it is assumed that the physical process of dissipation of flexural vibrational energy in a plate with discrete rubber inserts is determined by dry and viscoelastic friction between the elements of the structure under study.

2) It is assumed that the process of dissipation of vibrational energy in the plate under study is the same as in a system of excited plates with individual anti-vibration blocks located on its surface.

3) Based on the hypotheses and assumptions adopted in the research, a mathematical model was obtained, with the help of which it seems possible to reduce the level of sound radiation of plates with PVI. The mathematical model can be used as a tool for engineers and researchers to design and optimize systems with damping inserts to reduce sound radiation, which is important in various fields including sound protection and vibration isolation.

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